

Progression in Calculation: multiplication and division

Key language: share, equal groups, divide, divided by, half, multiply, divide, array, fact families, groups of, lots of, times, divide, share, remainder, factor, multiple, product, dividend, quotient, divisor, remainder

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication & Division: Calculations		calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals (=) signs Autumn 4 Spring 1	write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods Autumn 3 Spring 1	multiply two-digit and three-digit numbers by a one-digit number using formal written layout Spring 1	- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers - multiply and divide numbers mentally drawing upon known facts - divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context - multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 Autumn 4 Spring 1 Summer 1	- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication - divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context - divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context - perform mental calculations, including with mixed operations and large numbers Autumn 2

EYFS- reception early learning goal :*Children solve problems, including halving and sharing*

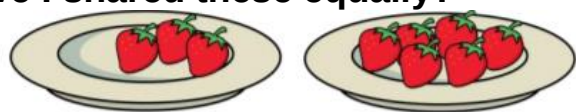
Teaching Point: Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Concrete

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.

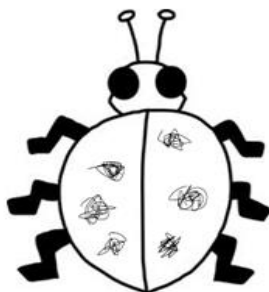


Have I shared these equally?



Provide opportunities for the children to share into equal groups.
For example, at snack time they could share bowls of grapes fairly between 3 or 4 children.
They could share out the cards or dominoes before playing a game.
Prompt the children to notice that sometimes they can make equal groups and sometimes they have items left over.

Pictorial



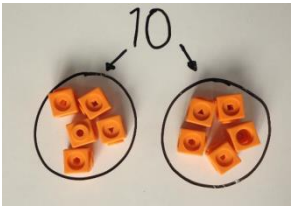
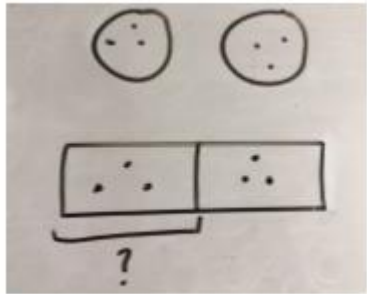
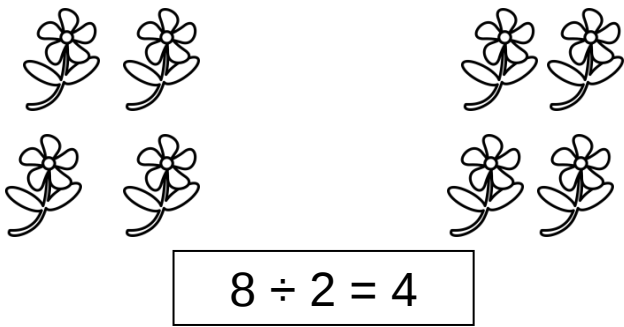
A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups

Abstract

Year 1: National Curriculum: Multiplication and Division -solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.

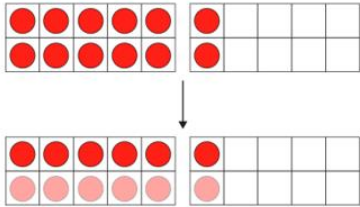
Objective and Strategies	Concrete	Pictorial	Abstract
Y1 Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 Groups? 	Represent the sharing pictorially.  Children use pictures or shapes to share quantities. 	Use the bar model $6 \div 2 = 3$  Share 9 buns between three people. $9 \div 3 = 3$

Year 2: National Curriculum: Multiplication and Division

calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals (=) signs

show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

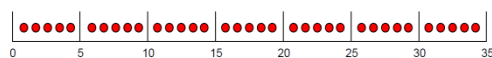
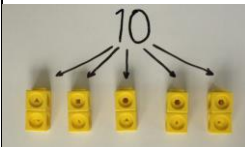
Objective and Strategies	Concrete	Pictorial	Abstract				
Y2 How to teach: Halving	Children to use counters to represent halving the total quantity   half of 12 = half of 10 + half of 2 = 5 + 1 = 6	'Emily has £20 in total, kept in two money-boxes. Each money-box contains the same amount of money. How much is in each money-box?'  2 × = 20 × 2 = 20 <table><tr><td colspan="2">20</td></tr><tr><td>?</td><td>?</td></tr></table>	20		?	?	half of 26 = half of 20 + half of 6 = 10 + 3 = 13 'Twenty-six is twenty plus six, so half of twenty-six is half of twenty plus half of six.'
20							
?	?						

Y2

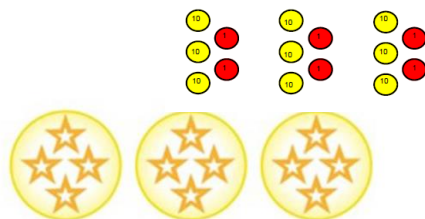
How to teach:

Division as grouping

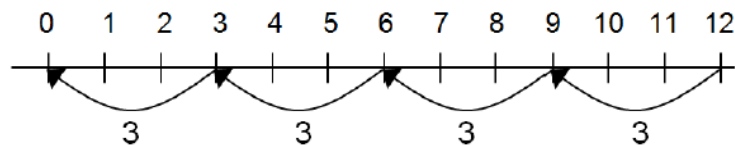
Divide quantities into equal groups.
Use cubes, counters, objects or place value counters to aid understanding.



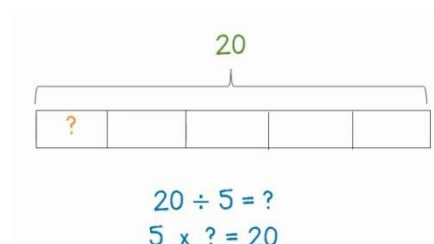
$$96 \div 3 = 32$$



Use a number line to show jumps in groups. The number of jumps equals the number of groups.

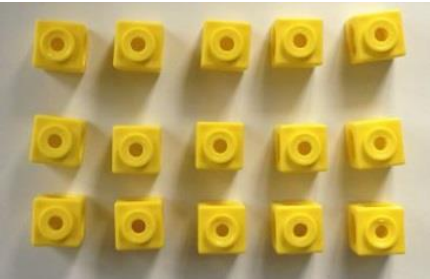
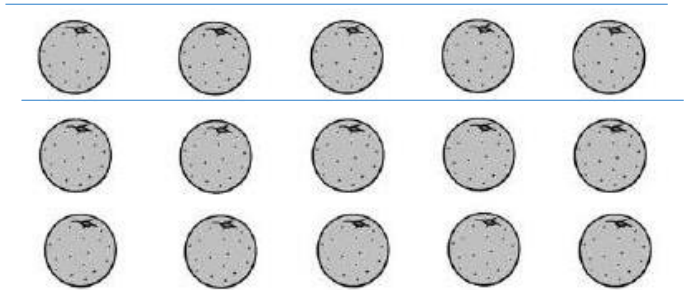


Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



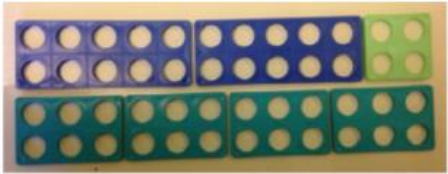
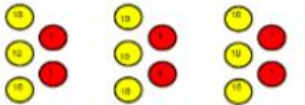
$$28 \div 7 = 4$$

Divide 28 into 7 groups.
How many are in each group?

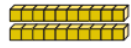
Objective and Strategies	Concrete	Pictorial	Abstract
Y2 How to teach: Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$

Y3- National Curriculum: Multiplication and Division write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2-digit numbers times 1-digit numbers, using mental and progressing to formal written methods

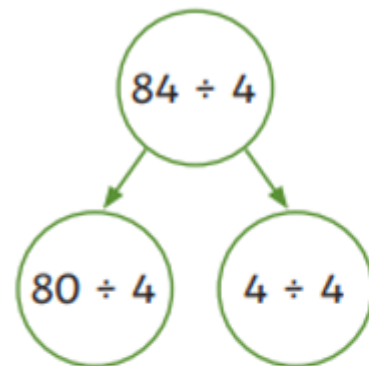
Teaching point: *It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.* Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

Objective and Strategies	Concrete	Pictorial	Abstract
Y3 How to teach: Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 $96 \div 3 = 32$ 	Continue to use bar modelling to aid solving division problems.  $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$

84 divided by 4

Tens	Ones
	
	
	
	

84 divided by 4

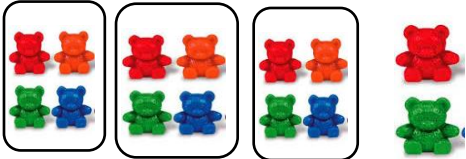
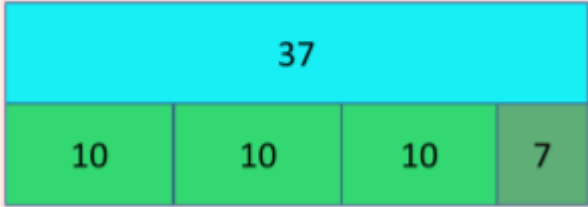


69 divided by 3

69		
23	23	23

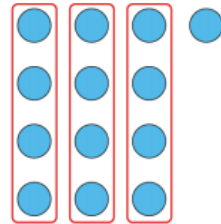
	2	1
4	8	4

Objective and Strategies	Concrete	Pictorial	Abstract
Y3 How to teach: Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Objective and Strategies	Concrete	Pictorial	Abstract
Y3 How to teach: Division with a remainder	$14 \div 3 =$ Divide objects between groups and see how much is left over  The image shows 14 teddy bears (7 red, 7 green) being divided into 3 groups. Each group contains 4 bears (2 red, 2 green), with 2 bears (1 red, 1 green) left over as the remainder.	Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g. $43 \div 8 =$  The image shows 43 loops grouped into 5 groups of 8 loops each, with 3 loops left over as the remainder. $43 \div 8 = 5 \text{ remainder } 3$ $37 \div 10 =$ Use bar models to show division with remainders.  The bar model shows a bar of length 37 divided into four sections: three sections of length 10 and one section of length 7, representing the remainder.	Complete written divisions and show the remainder using r. $29 \div 8 = 3 \text{ REMAINDER } 5$ ↑ ↑ ↑ ↑ dividend divisor quotient remainder

How many groups of 4 are in 13?

Dividend = 13, divisor = 4:



$$13 \div 4 = 3 \text{ r } 1$$

- 'Thirteen divided into groups of four is equal to three with a remainder of one.'

$$14 \div 3 =$$

Draw dots and group them to divide an amount and clearly show a remainder.



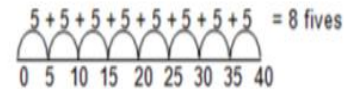
Use
line:

a number

Example without remainder:

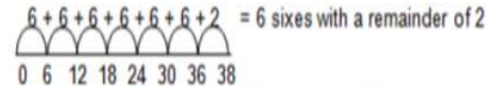
$$40 \div 5$$

Ask "How many 5s in 40?"



Example with remainder:

$$38 \div 6$$



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

If the children are ready they can be introduced to the vertical method of division .Please see the example below.

Y4- White Rose expectations – Autumn and Spring terms

Dividing a 2 digit number by a one digit number without a remainder

Children explore dividing 2-digit numbers by 1-digit numbers involving remainders.

Children apply their previous knowledge of dividing 2-digit numbers to divide a 3-digit number by a 1-digit number.

Teaching Point:

In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 – repeatedly subtracting individual groups of the divisor

Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

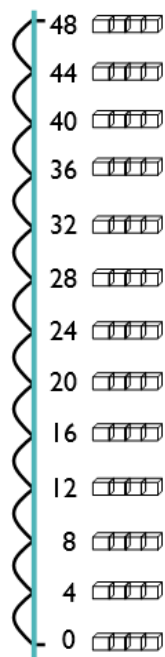
After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line

Y3/ Y4

How to teach:
Chunking
starting with
the vertical
method

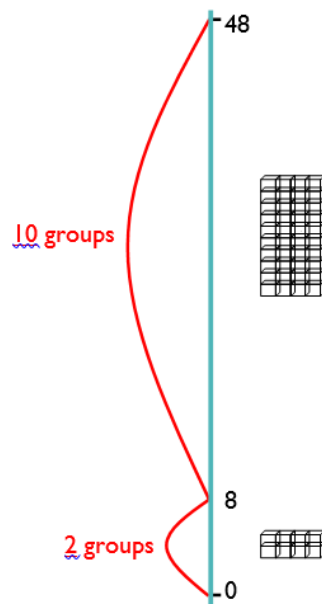
Step 1

$$48 \div 4 = 12 \text{ (groups of 4)}$$



Step 2

$$48 \div 4 = 10 \text{ (groups of 4)} + 2 \text{ (groups of 4)}$$
$$= 12 \text{ (groups of 4)}$$



The number line method can be linked to the chunking method to enable children to make links in their understanding.

Step 3

$$\begin{array}{r} 12 \\ 4 \overline{) 48} \\ \underline{- 40} \\ 8 \\ \underline{- 8} \\ 0 \end{array}$$

Answer: 12

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

Teaching Point:

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x

Y4

$$\underline{73} \div 3$$

$$\begin{array}{r} 24\text{r}1 \\ 3 \overline{) 73} \\ - 30 \\ \hline 43 \\ - 30 \\ \hline 13 \\ - 6 \\ \hline 7 \\ - 6 \\ \hline 1 \end{array}$$

Key facts box

1x	3
2x	6
5x	15
10x	30

$$\underline{196} \div 6$$

$$\begin{array}{r} 32\text{r}4 \\ 6 \overline{) 196} \\ - 120 \\ \hline 76 \\ - 60 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$$

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Y5- National Curriculum: Multiplication and Division

Divide numbers up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context

Teaching Point:

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. **During Year 5, children should be encouraged to be efficient when using the chunking method and not have any subtraction steps that repeat a previous step. For example, when performing $347 \div 8$ an initial subtraction of 160 (20×8) and a further subtraction of 160 (20×8) should be changed to a single subtraction of 320 (40×8).**

Y5

How to divide :a 3 and 4 digit number by a single digit, using chunking

$$\underline{523} \div 8$$

$$\begin{array}{r} 65\text{r}3 \\ 8 \overline{) 523} \\ - 320 \quad \text{40x} \\ \hline 203 \\ - 160 \quad \text{20x} \\ \hline 43 \\ - 40 \quad \text{5x} \\ \hline 3 \end{array}$$

$$2458 \div 7$$

$$\begin{array}{r} 351\text{r}1 \\ 7 \overline{) 2458} \\ - 2100 \quad \text{300x} \\ \hline 358 \\ - 350 \quad \text{50x} \\ \hline 8 \\ - 7 \quad \text{1x} \\ \hline 1 \end{array}$$

Y5 and
Y6

How to
divide:
using
the bus
stop
method

$$\begin{array}{r} \text{h t o} \\ 0 \ 4 \ 1 \ \text{R}1 \\ \hline 4 \) \ 1 \ 6 \ 5 \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0 \ 4 \ 0 \ 0 \ \text{R}7 \\ \hline 8 \) \ 3 \ 2 \ 0 \ 7 \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y5 and Y6

How to divide: using the bus stop method

Step 1

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \\ 7 \\ \underline{7} \\ 0 \end{array}$$

Step 2

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \\ 3 \\ \underline{3} \\ 0 \end{array}$$

Step 3

Move onto larger numbers

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \\ \underline{40} \\ 13 \\ \underline{16} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 4 \end{array}$$

Y6- National Curriculum: Multiplication and Division

divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Teaching Point: children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places.

Stage 1:

This should first be demonstrated using a simple calculation such as $13 \div 4$ to show the remainder initially as a fraction.



Using practical equipment, children can see that for $13 \div 4$, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

Step 2:

$$3574 \div 8$$

$$\begin{array}{r} 8 \overline{) 3574} \\ - 3200 \\ \hline 374 \\ - 320 \\ \hline 54 \\ - 48 \\ \hline 6 \end{array}$$

400x
40x
6x

$$\frac{6}{8}$$

remainder
divisor

So $3574 \div 8$ is $446\frac{6}{8}$
(when the remainder is shown as a fraction)

Long division

$432 \div 15$ becomes

$$\begin{array}{r} 28 \text{ r } 12 \\ 15 \overline{) 432} \\ \underline{30 } \\ 132 \\ \underline{120} \\ 12 \end{array}$$

$432 \div 15$ becomes

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30 } \quad 15 \times 20 \\ 132 \\ \underline{120} \quad 15 \times 8 \\ 12 \end{array}$$

$$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$$

$$\begin{array}{r}
 \mathbf{126} \\
 19 \overline{) 2394} \\
 \underline{19} \\
 49 \\
 \underline{38} \\
 114
 \end{array}$$

Ready Reckoner:

19
38
57
76
95
114
133
152
171
190

- 19 goes into 23 once. Take 19 from 23.
- Bring the 9 down.
- 19 goes into 49 twice. Take 38 from 49 to leave 11.
- Bring the 4 down.
- 19 goes into 114 six times.
- Answer is 126.

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25$, $\frac{3}{4} = 0.75$

Fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

Tenths: $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, $\frac{3}{10} = 0.3$, $\frac{4}{10} = 0.4$, $\frac{5}{10} = 0.5$, $\frac{6}{10} = 0.6$, $\frac{7}{10} = 0.7$, $\frac{8}{10} = 0.8$, $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, $3574 \div 8$, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms.

As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75.

