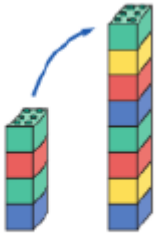
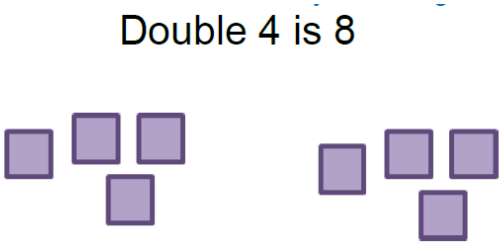
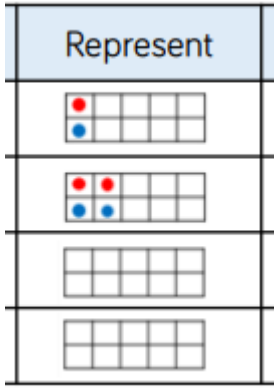
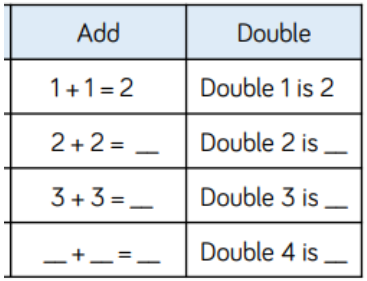
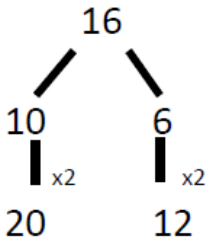
Progression in Calculation: multiplication and division

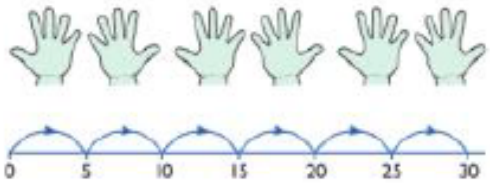
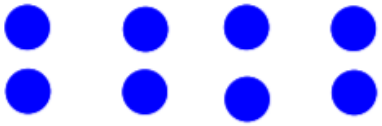
Key Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups._share, equal groups, divide, divided by, half, multiply, divide, array, fact families, groups of, lots of, times, divide, share, remainder, factor, multiple, product, dividend, quotient, divisor, remainder

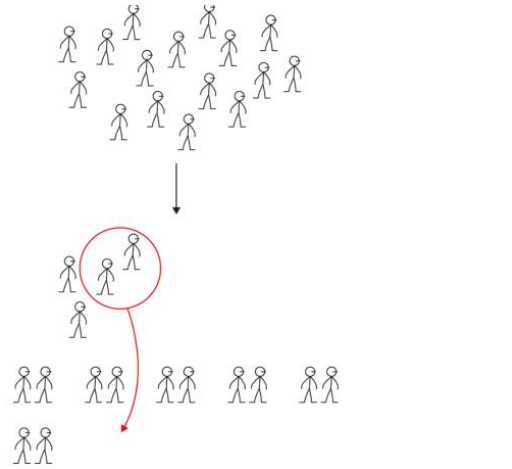
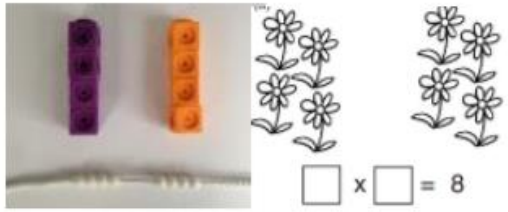
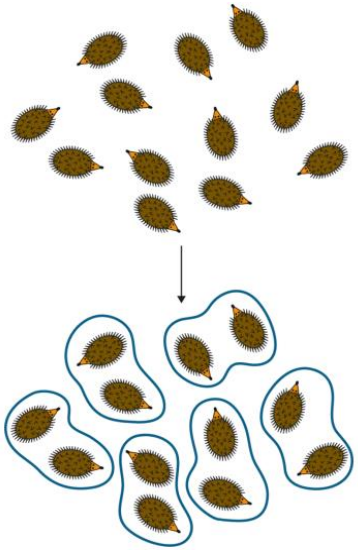
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication & Division: Calculations		• calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs **Autumn 4 Spring 1**	• write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods **Autumn 3 Spring 1**	• multiply two-digit and three-digit numbers by a one-digit number using formal written layout **Spring 1**	• multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 **Autumn 4 Spring 1 Summer 1**	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • perform mental calculations, including with mixed operations and large numbers **Autumn 2**

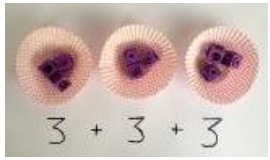
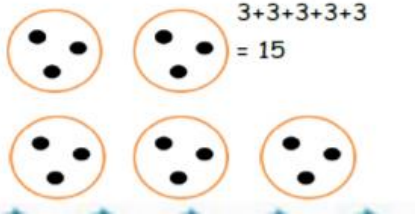
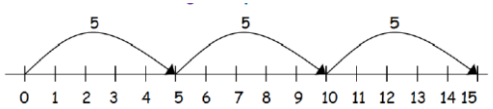
Multiplication

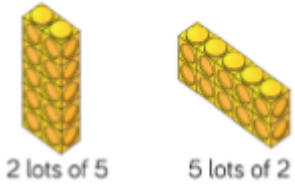
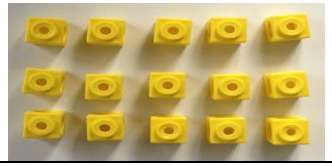
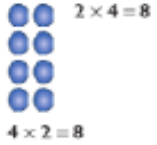
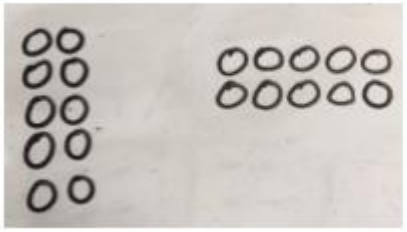
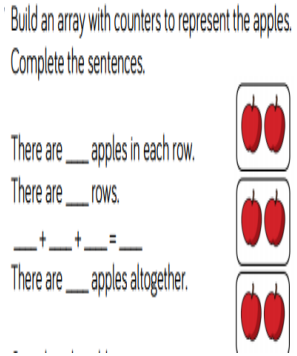
Year 1: National Curriculum: Multiplication and Division -solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.

Objective and Strategies	Concrete	Pictorial	Abstract
Year 1 Doubling	Use practical activities to show how to double a number.  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double a number.  Double 4 is 8  Represent	 Add / Double 1 + 1 = 2 / Double 1 is 2 2 + 2 = __ / Double 2 is __ 3 + 3 = __ / Double 3 is __ __ + __ = __ / Double 4 is __  16 10 x2 → 20 6 x2 → 12 Partition a number and then double each part before recombining it back together.

Objective and Strategies	Concrete	Pictorial	Abstract
Y1 **How to teach:** Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.  Children make representations to show counting in multiples. Skip counting in twos/counting even numbers – number line: 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Y1 Making equal groups	Using concrete resources, or working practically to count children, begin by presenting an even quantity of identical objects. As a class, first count in ones, moving each item from an uncounted area to a counted area as you go. Then count the objects again, this time counting in groups of two, moving *pairs* of items as you count.   Use manipulatives to create equal groups.	Once children can confidently 'pair off' concrete resources, progress to counting even quantities that are represented pictorially. Now encourage children to draw rings around each group of two to support their counting. 	2 + 2 + 2 + 2 + 2 = 5 + 5 + 5 = How many fish are there? Complete the sentences.  __ + __ + __ = __ There are ___ fish.

Objective and Strategies	Concrete	Pictorial	Abstract
Y1 **How to teach:** Repeated addition	   3 + 3 + 3 Use different objects to add equal groups.	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether?  3+3+3+3+3 = 15  5 + 5 + 5 = 15 There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

Objective and Strategies	Concrete	Pictorial	Abstract
Y1 **How to teach:** Arrays- showing commutative multiplication	Use arrays to illustrate commutativity . $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2 Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  $4 \times 2 = 8$ $2 \times 4 = 8$  $2 \times 4 = 8$ $4 \times 2 = 8$ Children to represent the arrays pictorially. 	 Build an array with counters to represent the apples. Complete the sentences. There are ___ apples in each row. There are ___ rows. ___+___+___=___ There are ___ apples altogether. Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

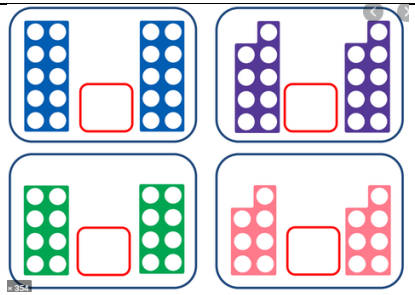
National Curriculum: Multiplication and Division - Year 2

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Objectives and strategies	Concrete	Pictorial	Abstract
Y2 How to teach: Equal groups	  	  5 / 5 / 5  *'There are four groups of two footballs.'* 2 / 2 / 2 / 2	3 x 2 = 6 factor x factor = product 6 = 3 x 2 product = factor x factor

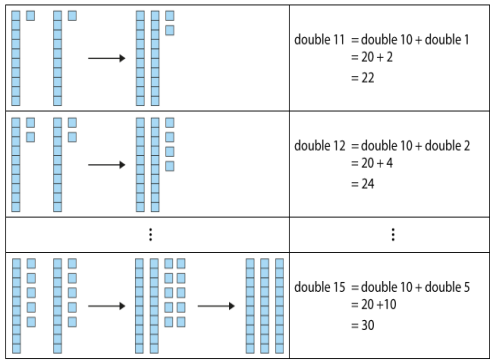
Y2

How to teach: Doubling 2 digit numbers



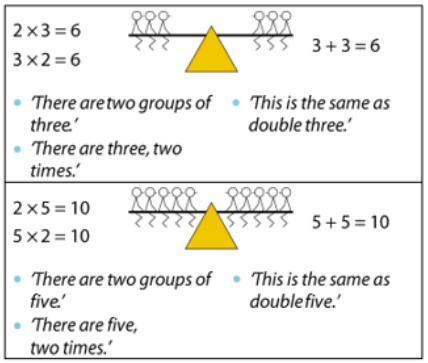
Model using base10 and place value counters

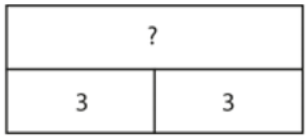
Doubling 11–15:



Doubling multiples of ten:

Draw pictures and representations to show how to double numbers.

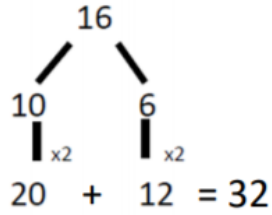




$2 \times 3 = \square$

- 'Two groups of three.'
- 'Three, two times.'
- 'Double three.'

Partition a number and then double each part before recombining it back together.



Part–part–whole diagrams:

'In all of these diagrams the parts are equal. Fill in the missing numbers.'

10	
?	?

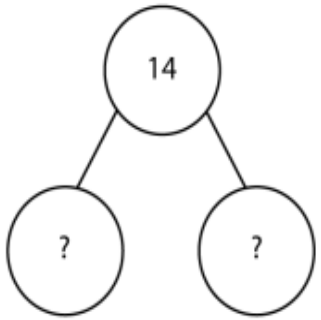
?	
10	10

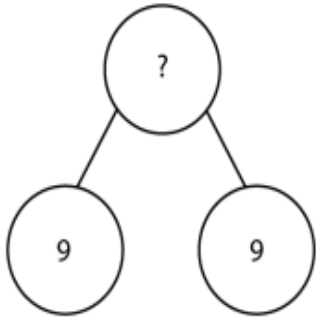
?	
7	7

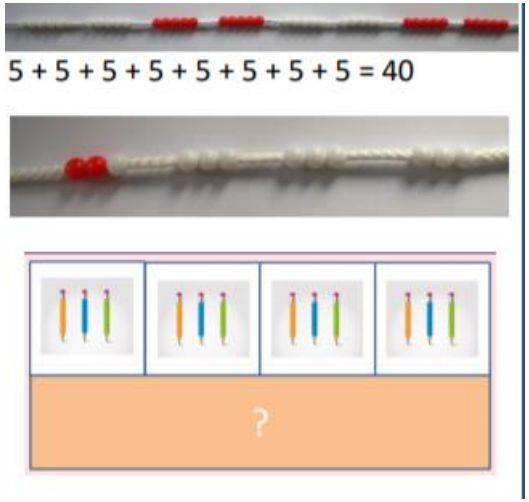
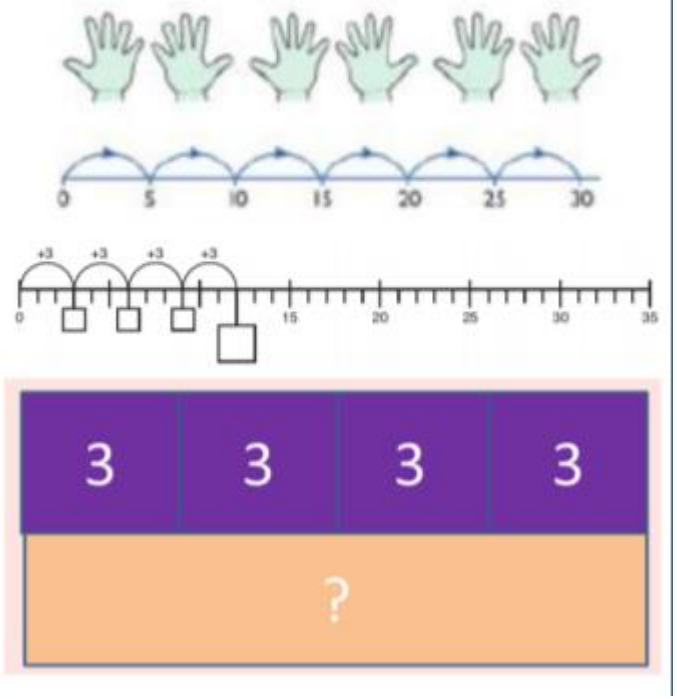
?	
3	3

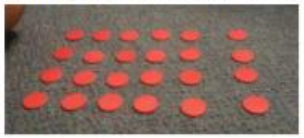
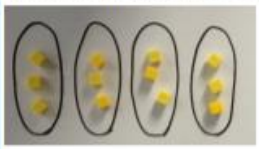
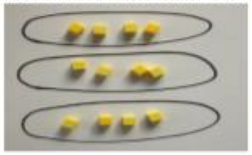
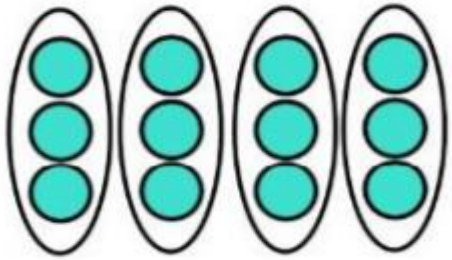
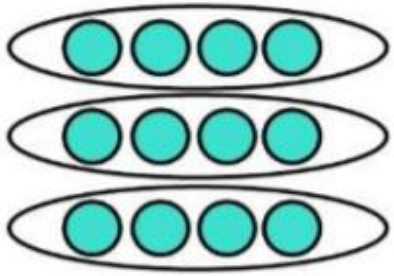
?	
?	6

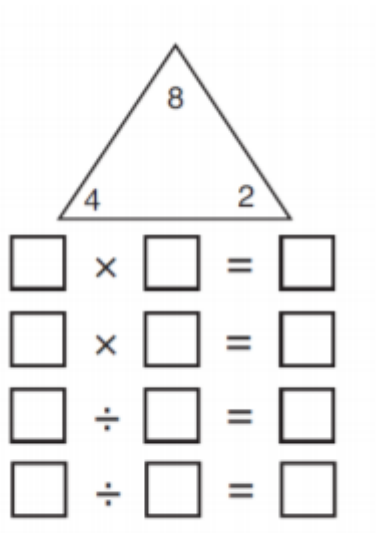
1000	
500	?





Objective and Strategies	Concrete	Pictorial	Abstract
Y2 **How to teach:** Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 	Number lines, counting sticks and bar models should be used to show repre-sentation of counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Objective and Strategies	Concrete	Pictorial	Abstract
Y2 **How to teach:** Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Objective and Strategies	Concrete	Pictorial	Abstract
Y2 **How to teach:** Using the Inverse : This should be taught alongside division, so pupils learn how they work alongside each other.			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

National Curriculum: Multiplication and Division - Year 3

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

TEACHING POINT

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method. It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

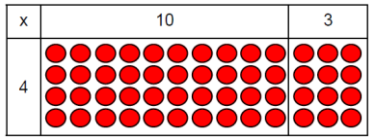
From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

Objective and Strategies	Concrete	Pictorial	Abstract

Y3

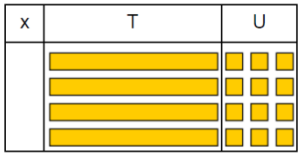
How to teach:
grid method for two digit x single digit

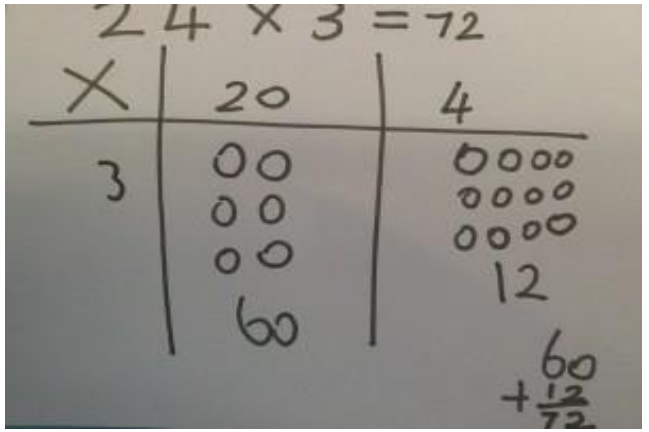
Show the link with arrays to first introduce the grid method. By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.
4 rows of 13





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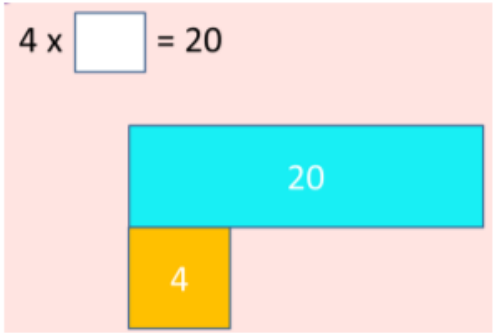
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

Bar model are used to explore missing numbers

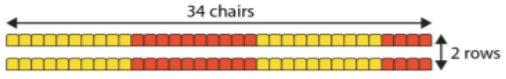
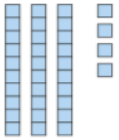
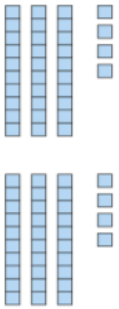


Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

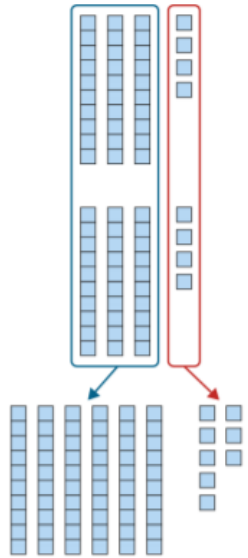
×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

Objective and Strategies	Concrete	Pictorial	Abstract
Y3 and Y4 **How to teach:** two digit times one digit using column method	*'There are two rows, each with thirty-four chairs. How many chairs are there altogether?'*  Step 1 – partition thirty-four:  34 = 30 + 4 • *'Thirty-four is equal to three tens and four ones.'* 34 = 3 tens + 4 ones Step 2 – make a copy of the representation (multiply by two):  34 × 2 • *'Three-tens-and-four-ones multiplied by two.'*	Work through some examples without base 10, until children can : 1. confidently partition the 2 digit number into tens and ones, 2. multiply the resulting part by the single digit. 3. Add the partial products *'One pack of biscuits costs 84p. How much do six packs cost?'* Example working 1: 84 × 6 = 504 80 4 80 × 6 = 480 4 × 6 = 24 480 + 24 = 504 *'Six packs of biscuits cost 504 p.'* Example working 2: 84 × 6 = 80 × 6 + 4 × 6 = 480 + 24 = 504 *'Six packs of biscuits cost 504 p.'*	Multiplication algorithm – expanded layout: Step 1 – write the factors: 10s 1s 3 4 x 2 Step 2 – multiply the single-digit number by the ones 10s 1s 3 4 x 2 8 2 × 4 ones = 8 ones Step 3 – multiply the single-digit number by the tens: 10s 1s 3 4 x 2 8 2 × 4 ones = 8 ones 6 0 2 × 3 tens = 6 tens Step 4 – add the partial products: 10s 1s 3 4 x 2 8 2 × 4 ones = 8 ones 6 0 2 × 3 tens = 6 tens 6 8

Step 3 – multiply the tens and ones and recombine:



$34 \times 2 = 30 \times 2 + 4 \times 2$
$= 60 + 8$
$= 68$

- *'Three-tens-and-four-ones multiplied by two is equal to three tens multiplied by two and four ones multiplied by two.'*

3 tens × 2 = 6 tens

4 ones × 2 = 8 ones

Now compare the expanded algorithm with the informal written method.

Comparing methods:

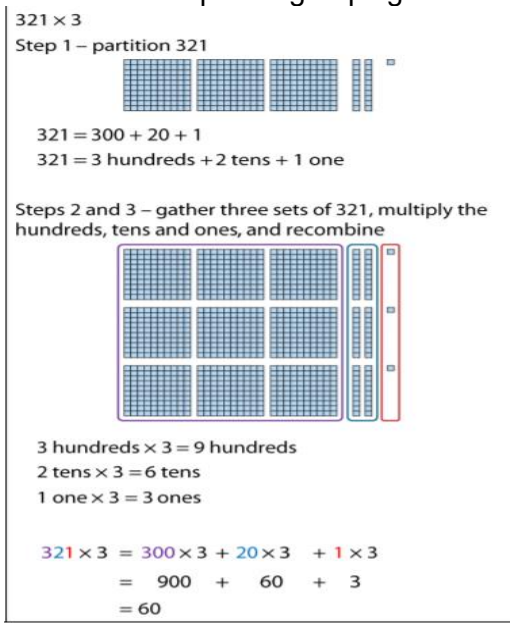
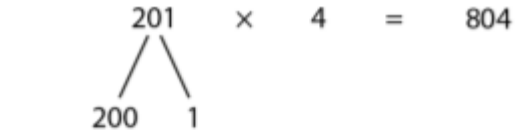
Informal written method:	Expanded multiplication algorithm:
$34 \times 2 = 30 \times 2 + 4 \times 2$ $= 60 + 8$ $= 68$	10s 1s 3 4 × 2 8 — 2 × 4 ones = 8 ones 6 0 — 2 × 3 tens = 6 tens 6 8

National Curriculum :Multiplication and Division-Y4

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Teaching point:

Children will move to year 4 using whichever method they were using as they transitioned from year 3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number. They will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number before moving onto the column method

Objective and Strategies	Concrete	Pictorial	Abstract
Y4/Y5 **How to teach:** Three digit numbers by a one digit number without regrouping . Column multiplication using base 10 and place value counters	Before using the short –multiplication algorithm to multiply 3 digit numbers by 1 digit numbers return to the use of base 10 to demonstrate that the partitioning approach already used for 2 digit numbers can be extended to 3 digit numbers. Begin with an example that doesn't require regrouping.  321 × 3 Step 1 – partition 321 321 = 300 + 20 + 1 321 = 3 hundreds + 2 tens + 1 one Steps 2 and 3 – gather three sets of 321, multiply the hundreds, tens and ones, and recombine 3 hundreds × 3 = 9 hundreds 2 tens × 3 = 6 tens 1 one × 3 = 3 ones 321 × 3 = 300 × 3 + 20 × 3 + 1 × 3 = 900 + 60 + 3 = 60	Children can use grid method along side the base 10 or partition the numbers as in the example below.  201 × 4 = 804 200 1 200 × 4 = 800 1 × 4 = 4 800 + 4 = 804 Example 2: 427 × 3 = 400 × 3 + 20 × 3 + 7 × 3 = 1200 + 60 + 21 = 1281	Multiplication algorithm – compact layout: 3 2 1 × 3 9 6 3

Place-value counter representation of 521 × 3:

Step 1 – partition 521:

521 = 500 + 20 + 1

521 = 5 hundreds + 2 tens + 1 one

Steps 2 and 3 – gather three sets of 521, multiply the hundreds, tens and ones and recombine:

5 hundreds × 3 = 15 hundreds

2 tens × 3 = 6 tens

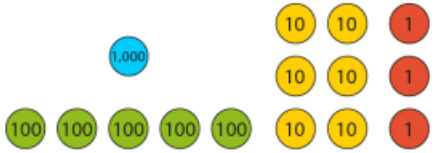
1 one × 3 = 3 ones

Step 4 - regroup the hundreds into thousands and hundreds

15 hundreds = 1 thousand + 5 hundreds

$521 \times 3 = 1000 + 500 + 60 + 3$

$= 1563$



Y4/Y5

How to teach:

Three digit numbers by a one digit number Column multiplication, with regrouping using base 10

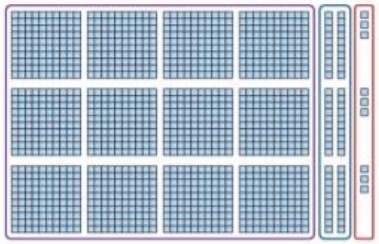
Dienes representation of 423 × 3:

Step 1 – partition 423

423 = 400 + 20 + 3

423 = 4 hundreds + 2 tens + 3 ones

Steps 2 and 3 – gather three sets of 423, multiply the hundreds, tens and ones and recombine



4 hundreds × 3 = 12 hundreds

2 tens × 3 = 6 tens

3 ones × 3 = 9 ones

$423 \times 3 = 400 \times 3 + 20 \times 3 + 3 \times 3$

$= 1200 + 60 + 9$

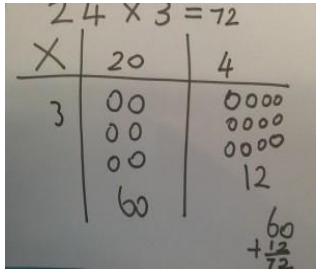
Step 4 - regroup the hundreds into thousands and hundreds

12 hundreds = 1 thousand + 2 hundreds

$423 \times 3 = 1000 + 200 + 60 + 9$

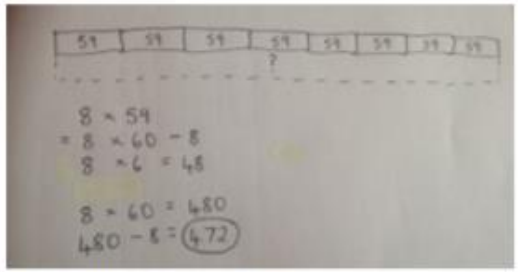
$= 1269$

.



×	300	20	7
4	1200	80	28

The grid method my be used to show how this relates to a formal written method.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

327

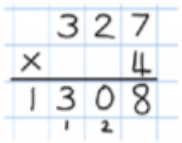
x 4

28

80

1200

1308



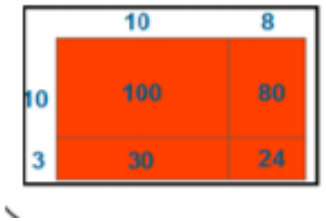
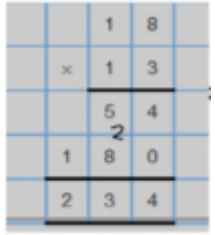
This may lead to a compact method.

Y4/Y5 Three digit numbers by a one digit number Column multiplication, with regrouping using place value counters.	Place-value counter representation of 521 × 3: Step 1 – partition 521: 521 = 500 + 20 + 1 521 = 5 hundreds + 2 tens + 1 one Steps 2 and 3 – gather three sets of 521, multiply the hundreds, tens and ones and recombine:  5 hundreds × 3 = 15 hundreds 2 tens × 3 = 6 tens 1 one × 3 = 3 ones $521 \times 3 = 500 \times 3 + 20 \times 3 + 1 \times 3$ $= 1500 + 60 + 1$ Step 4 - regroup the hundreds into thousands and hundreds 15 hundreds = 1 thousand + 5 hundreds $521 \times 3 = 1000 + 500 + 60 + 3$ $= 1563$  	As above	As above

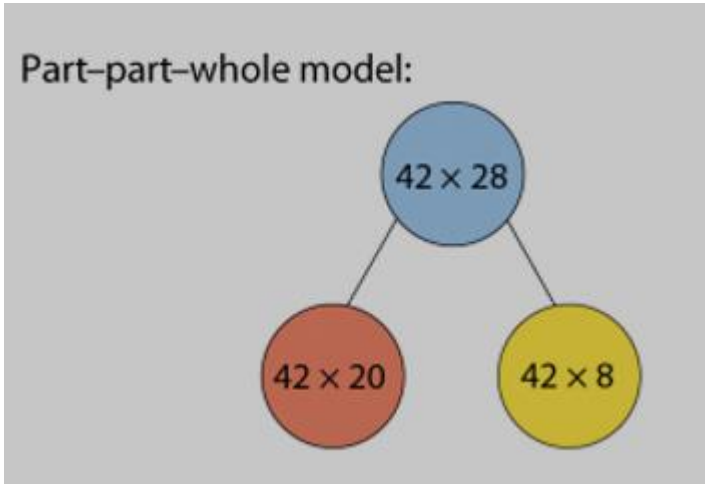
National Curriculum :Multiplication and Division-Y5

:multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Teaching point Children should only be expected to move towards the abstract method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

Objective and Strategies	Concrete	Pictorial	Abstract
Y5 and Y6 **How to teach column** multiplication compact method	Manipulatives such as place value counters and base 10 may still be used with the corresponding long multiplication modelled alongside as in the example above	Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit and numbers with up to four digits by a two-digit number 4346 x 8 Grid method can be used to show 2 digit x 2 digit 	4346 X 8 48 320 2400 32000 34768 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first 

x	4 000	300	40	6
8	32 000	2400	320	48

			Expanded Method: H T 0 2 7 × 3 4 2 8 (4 x 7) 8 0 (4 x 70) 2 1 0 (30 x 7) 6 0 0 (30 x 20) 9 1 8 Compact Method: H T 0 2 7 × 3 4 1 0 8 (4 x 27) 8 1 0 (30 x 27) 9 1 8 Step 1 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693 x 4) Step 2 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693 x 4) + 1 3 8 6 0 (693 x 20)

			Step 3 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693 x 4) + 1 3 8 6 0 (693 x 20) 1 6 6 3 2

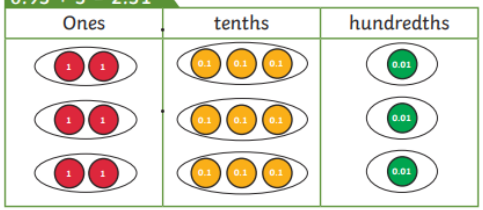
National Curriculum :Multiplication and Division-Y6

multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Teaching Point

By the end of year 6, children should be able to use the grid method and the compact method to multiply any number by a two-digit number. They could also develop the method to be able to multiply decimal numbers with up to two decimal places, but having been introduced to expanded and compact vertical methods in Year 5, it may be appropriate to use the expanded vertical method when introducing multiplication involving decimals.

Objective and Strategies	Concrete	Pictorial	Abstract
multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication Multiplying decimals up to 2 decimal places by a single digit.	Manipulatives such as place value counters and base 10 may still be used with the corresponding long multiplication modelled alongside as in the example above in Y4 and Y5.	x 2000 600 90 3 20 40000 12000 1800 60 = 53 860 4 8000 2400 360 12 = 10 772 + 64 632	2693 X 24

	6.93 ÷ 3 = 2.31 		4.92 × 3 T U . t h 4 . 9 2 x 3 0 . 0 6 (0.02 × 3) 2 . 7 (0.9 × 3) + 1 2 (4 × 3) 1 4 . 7 6 becomes T U . t h 4 . 9 2 x 3 1 4 . 7 6 2